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The proton's gluon distribution

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Abstract

The gluon distribution is dominated by the hard pomeron at small x and all Q^2 , with no soft-pomeron contribution. This describes well not only the DGLAP evolution of the hard-pomeron part of $F_2(x, Q^2)$, but also charm photoproduction and electroproduction, and is consistent with what is known about the longitudinal structure function.

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There is still no agreed fundamental explanation for HERA's striking discovery, the surprisingly large rise with increasing $1/x$ of the proton structure function $F_2(x, Q^2)$. This rise is seen to become more marked as Q^2 increases. The conventional view [1,2] is that the steeply-rising component is absent at small Q^2 and as Q^2 increases it is generated through pQCD evolution. We have argued [3] that this view is mathematically suspect, since it relies on an expansion of the splitting matrix which is likely to be unsafe [4] because it induces singularities that are almost certainly not present in the exact matrix. Instead, therefore, we maintain that a rapidly-rising term is present already at small Q^2 , so that pQCD evolution does not generate this term, but merely makes it become more prominent as Q^2 increases.

Our view is supported by ZEUS measurements [5] of the charm structure function $F_2^c(x, Q^2)$, which

already displays a strong rise with increasing $1/x$ even at small Q^2 . Fig. 1 shows the data at $Q^2 = 1.8 \text{ GeV}^2$; the lines are the conventional fit [1] to the data (MRST) and our own calculation. Indeed, HERA data for photoproduction and electroproduction of charm have the striking property [6] that at each fixed Q^2 they vary as the same power $x^{-\epsilon_0}$, with $\epsilon_0 \approx 0.4$. This behaviour is not widely appreciated as the data are normally shown on a log-linear plot rather than a log-log plot. By definition, we say that it is associated with the exchange of an object known as the hard pomeron [7]. A term with the same power is present also in the light-quark contribution to the proton structure function. To a good approximation, the hard-pomeron coupling to the charmed quark is found to have the same strength as to each of the light quarks. Once this assumption of flavour blindness is made [8] the hard-pomeron component of the complete structure function $F_2(x, Q^2)$ immediately provides a successful zero-parameter description of its

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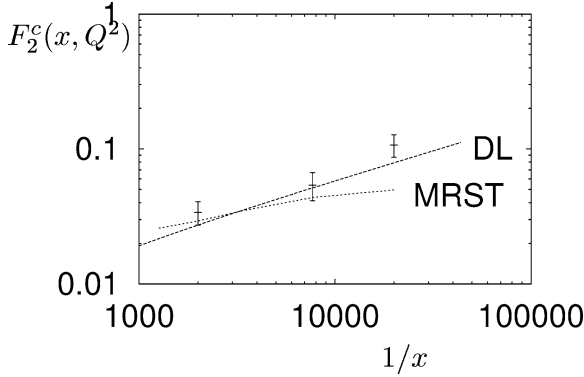


Fig. 1. Charm structure function data [5] at $Q^2 = 1.8 \text{ GeV}^2$ with curves from MRST [1] and [8] the form (1).

charm component $F_2^c(x, Q^2)$ at small x :

$$F_2^c(x, Q^2) = A_c(Q^2)^{1+\epsilon_0} (1 + Q^2/Q_0^2)^{-1-\epsilon_0/2} x^{-\epsilon_0} \quad (1)$$

with $Q_0 \approx 3 \text{ GeV}$ and $A_c \approx 6 \times 10^{-4}$. We define the charm cross section

$$\sigma^c(W) = \frac{4\pi^2\alpha_{\text{EM}}}{Q^2} F_2^c(x, Q^2) \Big|_{x=Q^2/(W^2+Q^2)}.$$

With (1)

$$\sigma^c(W) = 0.066 W^{2\epsilon_0} / (1 + Q^2/9.1)^{1+\epsilon_0/2}, \quad (2)$$

where the units are μb and GeV . This expression corresponds to the thin lines in the plots of Fig. 2.

These lines are not a fit to these data; they are calculated from the fit to the hard-pomeron component of the complete structure function $F_2(x, Q^2)$, making the flavour-blindness assumption.

The hard-pomeron contribution to the complete $F_2(x, Q^2)$ is the same, with A_c replaced with $A \approx 1.5 \times 10^{-3}$. We have shown [11] that DGLAP evolution, with a gluon structure function that is dominated at small x by hard-pomeron exchange alone, produces a Q^2 dependence for the hard-pomeron part of F_2 that agrees numerically with this. Our procedure, which can be applied only to the hard-pomeron component, gave almost identical outputs for LO and NLO evolution. A good numerical fit to the output of the DGLAP evolution for the small- x behaviour of the gluon structure function is

$$xg(x, Q^2) \sim 0.95(Q^2)^{1+\epsilon_0} (1 + Q^2/0.5)^{-1-\epsilon_0/2} x^{-\epsilon_0}. \quad (3)$$

This fit is valid for Q^2 between 5 and 500 GeV^2 .

We use the gluon distribution (3) to calculate charm production in leading-order pQCD and compare the result with (1). First, we calculate simple photon-gluon fusion in lowest order, for which the relevant equation [12] is (5.112) of the book by Roberts¹

$$F_2^c(x, Q^2) = 2e_c^2 \frac{\alpha_s(Q^2 + 4m_c^2)}{2\pi} \times \int_{ax}^1 dy g(y, Q^2 + 4m_c^2) f(x/y, Q^2), \quad (4a)$$

where

$$\begin{aligned} f(z, Q^2) &= v \left[4z^2(1-z) - \frac{1}{2}z - \frac{2m_c^2}{Q^2}z^2(1-z) \right] \\ &\quad + \left[\frac{1}{2}z - z^2(1-z) \right. \\ &\quad \left. + \frac{2m_c^2}{Q^2}z^2(1-3z) - \frac{4m_c^4}{Q^4}z^3 \right] L, \\ a &= 1 + 4m_c^2/Q^2, \\ v^2 &= 1 - 4m_c^2/[Q^2(y/x - 1)], \\ L &= \log\left(\frac{1+v}{1-v}\right). \end{aligned} \quad (4b)$$

For this leading-order calculation we again [11] set $\Lambda_{\text{QCD}} = 140 \text{ MeV}$. We have to choose the argument of α_s and the scale μ of the gluon structure function; physical intuition leads us to take $Q^2 + 4m_c^2$ for both, though it must be recognised that this is a mere guess. We need also to fix a value for m_c . We find that 1.3 GeV gives good results: the thick lines in Fig. 2 are the output of the calculation, while the thin lines are the phenomenological fit (2).

As we have said, our gluon distribution is hard-pomeron dominated; when we use it to calculate charm production we are modelling the strength of the coupling of the hard pomeron to the charm quark. As can be seen from the plots in Fig. 2, the calculation also includes threshold effects which make the rise steeper than $W^{2\epsilon_0}$ at small W . To a small extent,

¹ Note that the formula Roberts gives for v should be for v^2 .

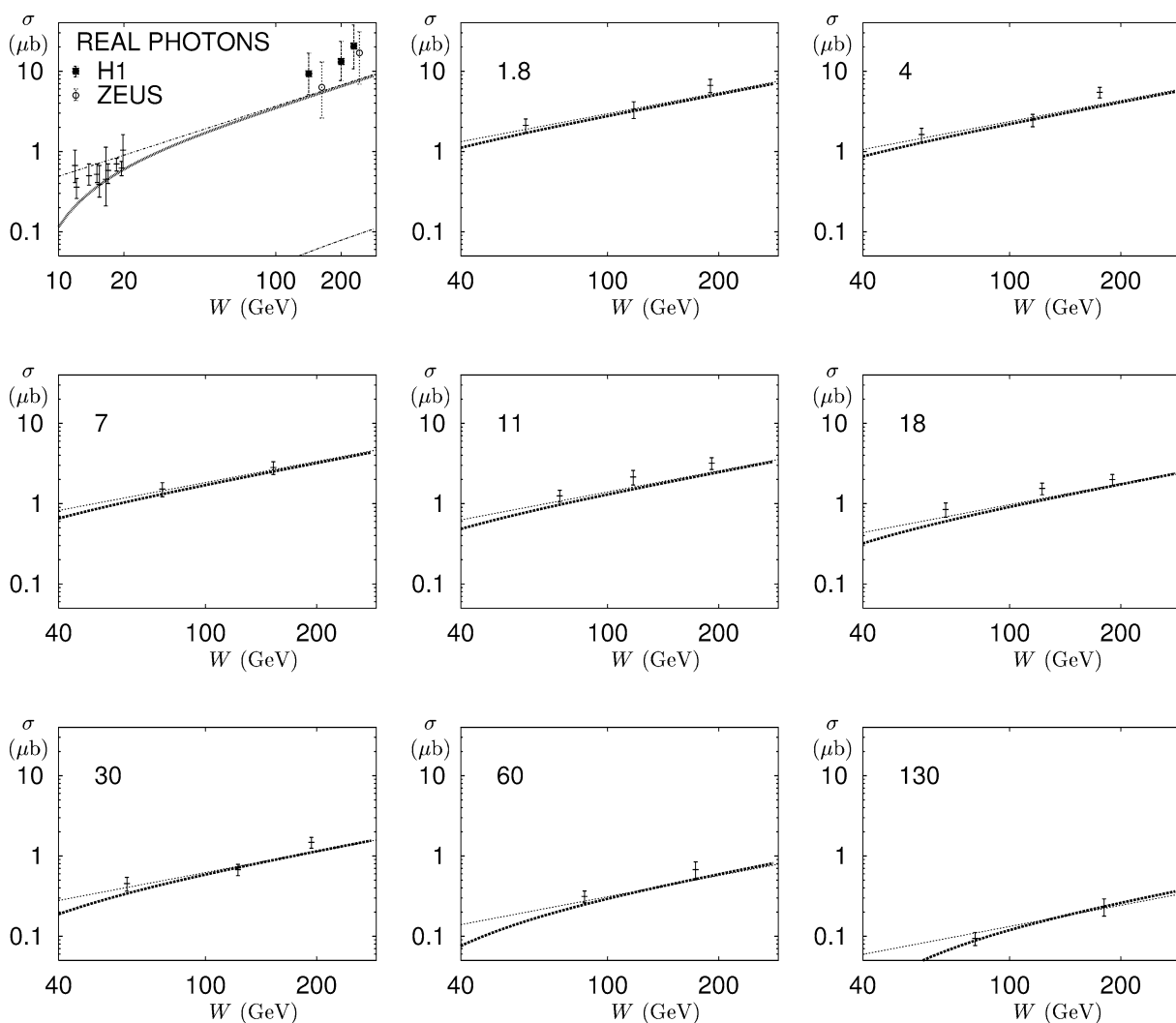


Fig. 2. Charm cross section: pQCD calculation (thick lines) and the form (2) (thin lines). The data for $Q^2 > 0$ are from ZEUS [5]. The photoproduction data are from H1 [9] and ZEUS [10], who give references to the fixed-target data. The line in the lower right-hand corner of the photoproduction plot is our calculation for b -quark production (for mass 4 GeV).

these threshold effects depend on the behaviour of the gluon distribution for values of x that are beyond the small- x region where (3) is valid. We have used (3) multiplied by $(1 - x)^n$ with $n = 5$; changing n by one unit changes the charm photoproduction cross section by less than 15% at $W = 10$ GeV and by 1% or less when $W > 50$ GeV.

At low Q^2 , and particularly for photoproduction, the magnitude of the cross section is sensitive to the value chosen for m_c . Changing m_c by 100 MeV away

from our preferred value of 1.3 GeV changes the photoproduction cross section by more than 20% at the higher energies. We have not included any possible contribution from the hadronic structure of the photon, because its magnitude is so uncertain. Our calculations are consistent with it being small, but this may not be true [13]. If indeed it is small, one needs [13] a structure function such as ours or the old MRSG [14] to reproduce the steep W -dependence of the data [13] at small Q^2 . The more modern MRST2001 [1] and

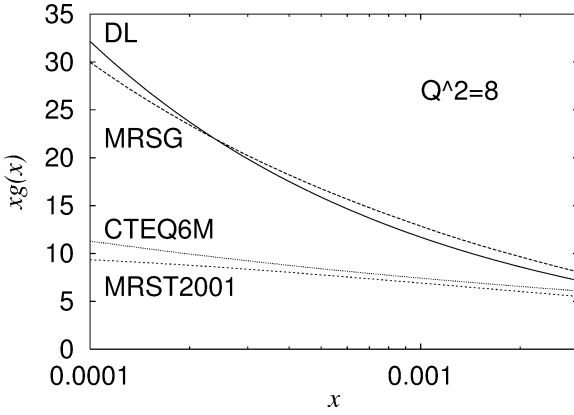


Fig. 3. Various gluon structure functions [15] at $Q^2 = 8 \text{ GeV}^2$.

CTEQ6M [2] gluon structure functions are not large enough, and not steep enough, to reproduce the low- Q^2 data, as is obvious from Figs. 1 and 3.

So, with $\mu^2 = Q^2 + 4m_c^2$ and $m_c = 1.3 \text{ GeV}$, leading-order photon-gluon fusion gives a good description of the data for F_2^c with hard-pomeron exchange alone, even down to $Q = 0$. At small enough x , the corresponding c -quark density is found to be almost identical with the hard-pomeron components of the densities of the light quarks. We previously [8] extracted the latter from the data for $F_2(x, Q^2)$ and [11] showed that, for Q^2 greater than about 5 GeV^2 , they agree very well with 4-flavour zero-mass DGLAP evolution. It is standard [16–18] that photon-gluon fusion at small Q^2 must be matched to DGLAP evolution at large Q^2 . Our calculation has this property: the two agree over a large range of Q^2 values.

We have not attempted to make a best fit to the charm-production data. In this, our approach is different from the so-called global fits [1,2]. Rather, our emphasis is on simplicity; we used only three free parameters in the fit to the small- x behaviour of $F_2(x, Q^2)$, and introduced no additional ones for $F_2^c(x, Q^2)$. The physics underlying our approach is very different from the conventional one and our successful application of pQCD to $F_2^c(x, Q^2)$ is a striking confirmation of the correctness of our extraction of the hard component from $F_2(x, Q^2)$. Most of $F_2(x, Q^2)$ at small x is the contribution from the soft component; this therefore plays the dominant role in the conventional approach, but it has not entered at all into the analysis we have described here.

The first plot in Fig. 2 shows also our calculation for the b -quark photoproduction cross section using $m_b = 4 \text{ GeV}$. It is not inconsistent with a measurement of H1 [19].

We now consider the proton's longitudinal structure function $F_L(x, Q^2)$. We calculate this in leading-order pQCD. The relevant equation is (5.110) of the book by Roberts [12]. However, we include the effect of the mass m_c in the contribution from c, \bar{c} ; this correction may be found in Eq. (E.3) of the review by Budnev et al. [20]. So

$$F_L(x, Q^2) = G(x, Q^2) + \frac{4\alpha_s(Q^2)}{3\pi} \int_x^1 \frac{dy}{y} \left(\frac{x}{y}\right)^2 F_2(y, Q^2), \quad (5)$$

where the contribution of the charm quark to $G(x, Q^2)$ is

$$G^c(x, Q^2) = 2e_c^2 \frac{\alpha_s(Q^2 + 4m_c^2)}{\pi} \times \int_{xa}^1 dy \left(\frac{x}{y}\right)^2 \left[\left(1 - \frac{x}{y}\right)v - \frac{2m_c^2 x}{Q^2 y} L \right] \times g(y, Q^2 + 4m_c^2) \quad (6)$$

with a , v and L defined in (4b). We have again had to choose the argument of α_s and the Q^2 -scale of the gluon structure function. We have made the same choice as before: $Q^2 + 4m_c^2$ for each. Again this is a guess and the output is sensitive to it at low Q^2 . The light quarks contribute similarly, with m_c replaced with 0.

The HERA experiments measure the reduced cross section

$$\sigma^r(x, y, Q^2) = F_2(x, Q^2) - \frac{y}{1 + (1 - y)^2} F_L(x, Q^2). \quad (7)$$

We have calculated this from our fit to F_2 and our gluon structure function (3); the results are shown in Fig. 4. The ranges of x and Q^2 shown are chosen because our fit to F_2 used only small- x data and because we found [11] that perturbative evolution only described it well for Q^2 greater than about 5 GeV^2 , so our gluon distribution is not reliable for smaller values. In Fig. 5 we compare our calculated F_L with the values

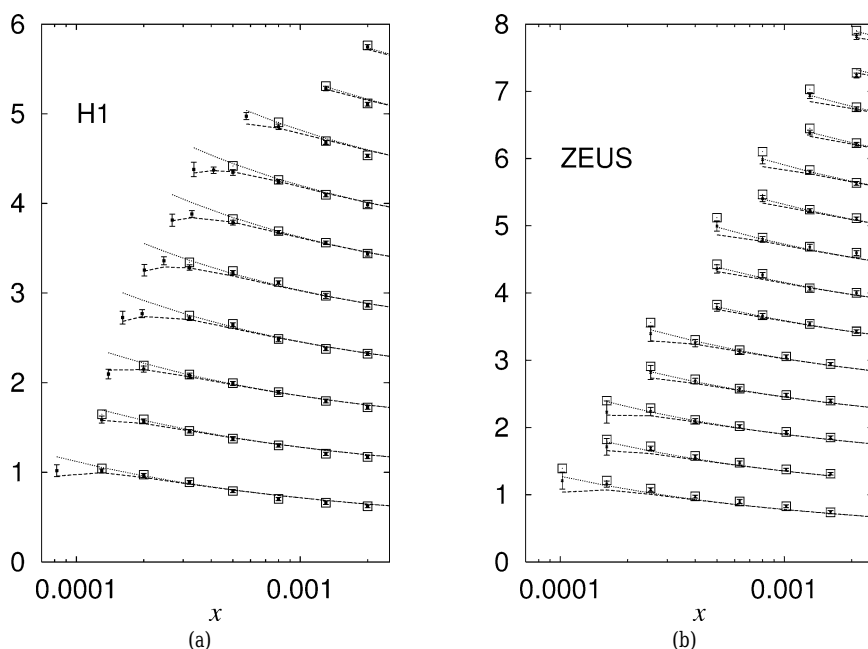


Fig. 4. H1 [21] and ZEUS [22] data for the reduced cross section (7). The open squares are the values of $F_2(x, Q^2)$ extracted from these data by the two experiments. The data in (a) range from $Q^2 = 5 \text{ GeV}^2$ at the bottom to 60 at the top and in (b) from 6.5 to 120; in each case the data are separated by adding an extra 0.5 at successive values of Q^2 . At each Q^2 the upper line is our fit [8] to $F_2(x, Q^2)$ (the ZEUS data in (b) were not available when the fit was made) and the lower line is the calculated reduced cross section.

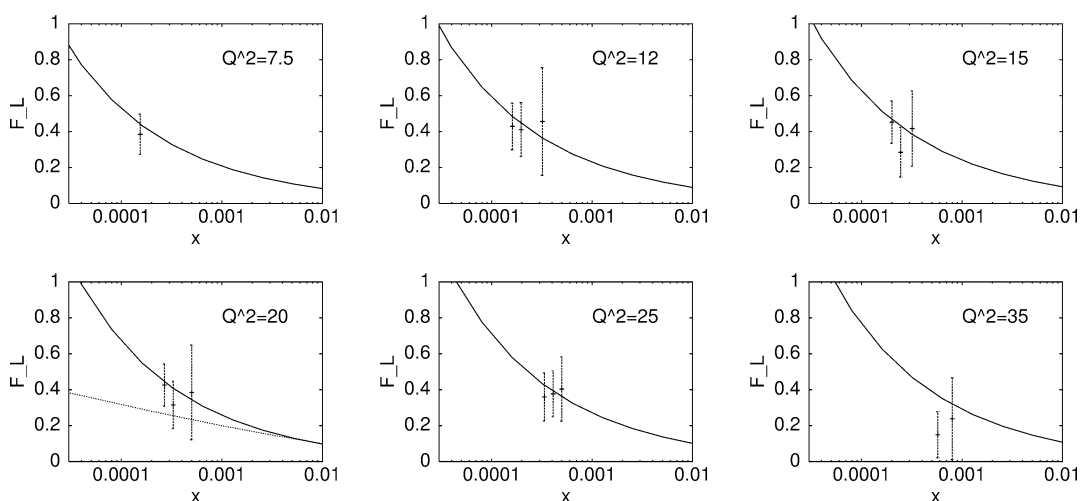


Fig. 5. H1 data [21] for the longitudinal structure function at various Q^2 values, with our pQCD calculations. The lower line on the $Q^2 = 20 \text{ GeV}^2$ plot is the MRST2001 prediction [1].

extracted by H1 [21] from their data. Separation of F_L from F_2 requires extrapolation and depends on some assumed parametrisation.

In conclusion, in this Letter we have continued our programme of reconciling the Regge and pQCD-evolution approaches to structure function data. We

use two sets of data, the charm structure function and the longitudinal structure function. In our calculation of F_2^c photon–gluon fusion at small Q^2 is matched to DGLAP evolution at large Q^2 . Our success in describing the charm and longitudinal structure functions provides further confirmation for the correctness of the two approaches and our understanding of how they fit together. We should warn, however, that in each case the extraction of the data from the raw measurements requires very large extrapolations. As MRST have observed [1], it would be particularly useful to have good data for $F_L(x, Q^2)$, since this offers rather direct information about the gluon distribution.

Our gluon structure function is larger and steeper at small values of x than is conventionally believed [1,2], particularly at small Q^2 , but we have shown in this Letter that there is experimental support for it. A less-steep gluon distribution will not explain the charm data at small Q^2 . Because our evolution procedure does not introduce spurious singularities at $N = 0$ into the splitting matrix, we conclude that the gluon distribution is larger than is usual in order to achieve the observed evolution of $F_2(x, Q^2)$.

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References

- [1] A.D. Martin, R.G. Roberts, W.J. Stirling, R.S. Thorne, Eur. Phys. J. C 23 (2002) 73.
- [2] CTEQ Collaboration, J. Pumplin, et al., JHEP 0207 (2002) 012.
- [3] J.R. Cudell, A. Donnachie, P.V. Landshoff, Phys. Lett. B 448 (1999) 281.
- [4] S. Catani, F. Hautmann, Nucl. Phys. B 427 (1994) 475.
- [5] ZEUS Collaboration, J. Breitweg, et al., Eur. Phys. J. C 12 (2000) 35.
- [6] A. Donnachie, P.V. Landshoff, Phys. Lett. B 470 (1999) 243.
- [7] A. Donnachie, H.G. Dosch, P.V. Landshoff, O. Nachtmann, Pomeron Physics and QCD, Cambridge Univ. Press, 2002.
- [8] A. Donnachie, P.V. Landshoff, Phys. Lett. B 518 (2001) 63.
- [9] H1 Collaboration, S. Aid, et al., Nucl. Phys. B 472 (1996) 32.
- [10] ZEUS Collaboration, M. Derrick, et al., Phys. Lett. B 349 (1995) 225.
- [11] A. Donnachie, P.V. Landshoff, Phys. Lett. B 533 (2002) 277.
- [12] R.G. Roberts, The Structure of the Proton, Cambridge Univ. Press, 1990.
- [13] S. Frixione, et al., Adv. Ser. Direct High Energy Phys. 15 (1998) 609, hep-ph/9702287.
- [14] A.D. Martin, R.G. Roberts, W.J. Stirling, Phys. Lett. B 354 (1995) 155.
- [15] Durham data base, <http://cpt19.dur.ac.uk/hepdata/pdf3.html>.
- [16] M.A.G. Aivazis, J.C. Collins, F.I. Olness, W.-K. Tung, Phys. Rev. D 50 (1994) 3102.
- [17] R.S. Thorne, R.G. Roberts, Phys. Lett. B 421 (1998) 303.
- [18] M. Buza, Y. Matiounine, J. Smith, W.L. van Neerven, Eur. Phys. J. C 1 (1998) 301.
- [19] H1 Collaboration, C. Adloff, et al., Phys. Lett. B 467 (1999) 156.
- [20] V.M. Budnev, et al., Phys. Rep. C 15 (1974) 181.
- [21] H1 Collaboration, C. Adloff, et al., Eur. Phys. J. C 21 (2001) 33.
- [22] ZEUS Collaboration, S. Chekanov, et al., Eur. Phys. J. C 21 (2001) 443.